Introduction:

The general picture on the ground state of many-body electron systems is based on free fermion systems, which are characterized by the Fermi surface (FS). Its presence continues even when we turn on small interactions or place ionic lattice: although FS deforms and individual particles change their mass and charge, such quasi-particles are adiabatically connected to the original electrons. In this case the properties of many-body system are reduced to those of individual particles and therefore the theory of many-body is essentially the same as the theory of conventional particle physics. This is the Fermi-liquid theory of Landau whose vast regime of validity was understood by the symmetry principle and renormalization group idea: most of the interactions become irrelevant in low energy and the system is controlled by the symmetry of the order parameter. Only in the special configurations of measure zero, there are instabilities which lead to new ground states. BCS and charge/spin density wave are some of them.

Strongly correlated system:

When the interaction is strong, however, the landscape of ground states is completely different. Particles that are deep inside the Fermi sea can be excited so that the Fermi surface does not have much meaning if there is such thing at all. Furthermore, if one particle moves, others have to follow due to the strong interaction, hence small
excitations are hard to be created if all lattice sites are uniformly occupied. This is the Mott gap. In one electron language, which is not proper in strongly interacting electrons, it is the band gap generated by the electron-electron interaction. A small change of parameters can induce big effects in principle and the phase diagram is more complicated, and provide a main challenge of 21st Century physics. See figure 1.

The Hubbard model describes the qualitative physics very concisely with just two parameters. We would understand the nature of high Tc superconductivity by solving it in two spatial dimensions. Unfortunately that was achieved neither exactly nor perturbatively in spite of the desperate efforts of many people for many decades. Even numerical approach is forbidden due to the sign problem.

**The origin of the difficulty:**

The Problem is the strong nature of the interaction. At this point, some of my high energy physics friends may wonder why electrons whose coupling constant is less than 1/100, interact strongly. The answer is the slowness of the electron. If the interaction is given by the local field theory with small coupling, the dynamics is described by the Feynman diagrams whose components are free propagators connected by interaction vertices. The issue of the convergence is matter of coupling. However, for non-relativistic particles under the static force, it is not just matter of coupling.

According to the Lippmann-Schwinger equation, the effective expansion parameter is the ratio of potential and kinetic energy. Therefore for slow enough electrons, it can be much bigger than one in spite of the small coupling constant of electromagnetic interaction. Electrons in 3d or 4f states are highly localized and therefore they can move only slowly from one lattice to the others. This is why electrons in transition metals are strongly correlated. So are they in lanthanoids and rare earth materials.

**Gauge/gravity duality:**

Recently, string theory found an exact duality between strongly interacting super-symmetric gauge theory and weakly interacting gravity theory in one higher dimension [1,2,3]. It is called gauge/gravity duality. Since the extra dimension can be understood as a scale parameter, one may figure the 5 dimensional theory as the collective description of all scale and Einstein equation encodes the information.

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**FIG. 1** Phase diagram of High Tc superconductor. (From Royal Society publishing, D. Galanakis et.al.)
of renormalization group. (see figure 2). The highlight of the duality is that it enables us to calculate all Green functions exactly. It would be desirable to find such duality in strongly interacting electron systems, which would provide a similar level of calculability. Calculating two point functions is enough to generate physical quantities that can be compared with experimental data. However, at this moment, such duality is far from obvious. Finding a gravity dual of given Hamiltonian is very hard if not impossible. Instead, one may postulate the following calculational scheme.

1. Identify operators creating the relevant degree of freedom. Find their spin and scaling dimensions. Introduce the source fields and extend them to higher dimensions.
2. Write down the Lagrangian of the source fields in the curved space-time.
3. If necessary, deform the metric to one other than AdS.
4. Assign boundary conditions for various fields.
5. Solve the equation of motion and calculate the action as a function of the boundary values.
6. Read off the Green functions by taking the derivatives of the action with respect to the boundary values. The Green functions generate the AC/DC transport coefficients or spectral function.

The strategy is to compare the calculated result with experiments. This is far from logically satisfactory theory. However, physics need jump to make meaningful progress. It is somewhat analogous to the early days of quantum mechanics. We have set of rules to compute but not much understanding. Think about the Schroedinger equation (SE) with potential. For a free particle, we can ‘derive’ SE from the dispersion relation and wave–particle duality. However, for the case with potential, SE is just a postulate which can be justified only by experiment. Today’s holography is similar in character. In fact many string theorists practices such scheme which is now dubbed as AdS/CMT.

**Present status:**

What has been achieved by such scheme so far? I can list very biased topics.
1. General formalism to construct finite temperature retarded Green functions and transport coefficients [4].

2. Mean field theory of superconductivity with s-wave condensation without explicit Higgs potential [5]. Similarly models with p-wave [6] as well as d-wave [7] condensation were constructed.

3. Models generating the resistivity linear in temperature [8,9,10].

4. Gravity dual of fluid [11,12,13].

5. Models showing metal-insulate transition generated by interaction. It is probed by the behavior of AC conductivity [14].

6. Transport near quantum critical point [15]

7. Models with Fermion coupling that induces Mott gap [16].

8. Understanding easy thermalization in strong coupling [17]. See figure 3.

9. Holography of Non-Fermi Liquid [18,19,20]

We can design models to solve any of the mysterious features of strongly correlated electron systems. But we still do not have a model having all the properties of high-Tc material.

This is a very rapidly developing field and I hope we can hear good news soon. See [21] for a review. With some luck, we will have a calculable theory of strongly interacting many-body systems soon.

![Image](image.png)

**FIG. 3** Any shape of shell falls to form a black hole in one dynamical time. This special dynamics in AdS is the mechanism for easy thermalization for strong coupling [17].

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**Reference**


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