Black holes in all

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Black holes are the most elementary and fascinating objects of General Relativity (GR). The fact that the effects of the space-time curvature are dramatic in their presence explains why it is relevant studying these systems. Recent developments in String/M Theory –expected to be a truly unified theory of all interactions– and in the application to the study of strongly coupled field theories via the AdS/CFT correspondence –stating that a gravitational solution in the bulk is dual to a non-gravitational conformal field theory– have prompted our interest in black holes in more than four space-time dimensions. Bearing in mind the transcendental impact of the connections between GR and other non-gravitational theories, we will describe here the different species of black holes in D space-time dimensions.

It is now commonly accepted that black holes (in four dimensions of space-time) are not only an intellectual possibility but do indeed exist in our Universe, usually lying at the centre of galaxies, including the Milky Way. The actual formation of objects with such special properties remains a quiz, but they are believed to correspond to the final stage in the evolution of a star that undergoes a catastrophic, self-gravitational collapse [1]. Stars usually rotate and the angular momentum ought to be preserved along the collapsing process, so the most common black hole formed this way should be a rotating black hole. These black holes have their theoretical realization in the rotating black hole solution of Roy Kerr [2], a description providing an absolutely exact representation of all the black holes that exist. The black hole solution of Kerr is characterized only by two parameters (called charges in this context): mass and angular momentum, something colloquially referred to as that black holes have no hair. Conversely, according to uniqueness theorems, the most general four dimensional, neutral, asymptotically flat, stationary black hole is Kerr's. These objects have a theoretical counterpart in higher space-time dimensions. Unlike in four, in higher dimensions *there is more room* and black hole solutions display richer features. In fact, the four-dimensional uniqueness theorems break down for $D > 4$.

The first black holes

In the 18th century it was already proposed [3] that very massive celestial objects could deflect light and drag it back due to self-gravitational effects. In 1916, a gravitational object with these characteristics was shown by Karl Schwarzschild [4] to be possible within the framework of General Relativity. Schwarzschild's solution is indeed considered the first rigorous example of a black hole: a neutral (electrically uncharged), static, asymptotically flat solution of Einstein's equation in four dimensions of space-time, displaying a horizon of spherical topology.

The name black hole was coined by John Archibald Wheeler in 1967 and prevailed despite the strong opposition of influential physicists such as Richard Feynman.

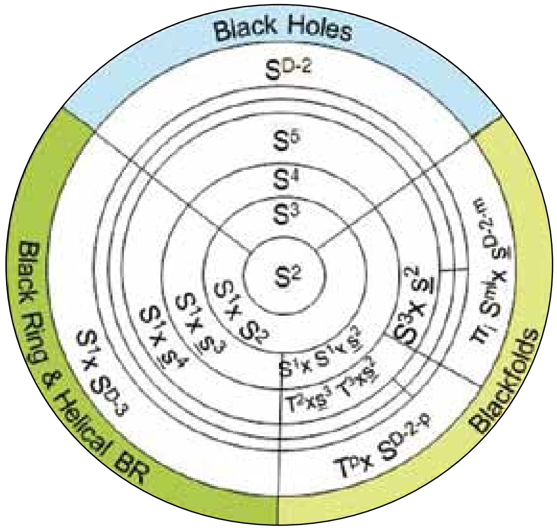

Figure 1. Summary of neutral stationary unit horizen black hole solutions in D≥4 dimensions classified by its horizon topologies. The four dimensional spherical S^2 Kerr black hole in the center. Each of the outer shells represent one higher dimension i.e. in D=5, the first outer shell from the center, there are S^3 and $S^1 \times S^2$ topologies of the event horizons corresponding to the Myers-Perry black hole and the black ring (BR) and helical BR.

A classification of higher dimensional black holes

Within the solid framework of D-dimensional GR, black holes have an event horizon. This is what distinguishes them from any other stellar objects that lack it. The event horizon is a boundary of space-time beyond which events cannot affect the outside region – the outgoing light will be dragged inwards giving its name to the black holes. The topology of this frontier characterizes and sets a classification of the black objects in any dimensions.

On the one hand, the classification of neutral, asymptotically flat, *static* black holes (non-rotating solutions with null Killing vector fields on the horizon) is simple and complete. The Schwarzschild-Tangherlini black hole has been proved to be the only allowed static black hole in all dimensions D > 3, and the existence of static black holes with non-spherical S^{D-2} topologies is accordingly ruled out.

In contrast, *stationary* black holes (those with intrinsic rotation), can give rise to event horizons with more sophisticated topologies. The current status of the classification of stationary black holes by horizon topologies is far from complete, and most of the higher-dimensional black hole solutions allowed in principle remain unknown. Let us first review what the situation is for stationary, asymptotically flat black holes in all dimensions (see Fig. 1 for a quick summary).

- D≤4. There are no asymptotically flat black holes below four dimensions. The lowest dimensional known black hole is the well-known Kerr black hole in four dimensions. As we previously discussed, this spherical (S^2) rotating black hole is the only one in D=4, and is uniquely characterized by its conserved charges.

- D=5. In five dimensions the situation is different since there are no equivalent general uniqueness theorems. Now the possible topologies of the event horizon are not only S^3, but also $S^1 \times S^2$. The Myers-Perry black hole solution (the extension of the Kerr rotating black hole to higher dimensions) corresponds to the former, S^3, case. The latter possibility, $S^1 \times S^2$, is also realized in the black ring of Emparan and Reall [5], with one angular momentum, and that of Pomeransky and Sen'kov [6], with two angular momenta in orthogonal planes. These are, in fact, the only possibilities (aside from the Lens L(p, q) topology where no explicit solution is known), according to rigidity theorems [7, 8]. All higher-dimensional black holes, regardless of their asymptotics, have been argued to be axisymmetric, that is, to display an axial U(1) spatial symmetry [9]. Evidence for the existence of black hole solutions with exactly one spatial U(1) was provided in [10] and dubbed helical black rings.
- D=6. When going one dimension further up, to six dimensions, the territory becomes vast and not many solutions are known. From cobordism theories, restrictions in the type of allowed topologies leave the open possibilities just to the following: S^4, $S^1 \times S^3$ and $S^2 \times \Sigma_g$ where Σ_g is a genus g Riemann surface (for example the S^2, with g = 0). For several years, the only known black hole solution in D=6 was, again, the MP black hole with two rotational symmetries and an S^4 event horizon. The more extravagant topologies $S^1 \times S^3$ were recently shown to be realized in the D=6, the thin black rings [11] (in the weak gravity approximation, for which self-gravitational effects are absent). There is also evidence of the existence of helical black rings and black 2-tuboids with $T^2 \times S^2$ horizons topologies, representing a particular type of blackfold in six dimensions. Explicit D=6 black hole metrics realizing the remaining possibility, $S^2 \times S^2$ for the event horizon, are unknown.
- D>6. In this case there are essentially no restrictions on the possible topologies. In fact, there are no analog rigidity theorems to restrict the topologies of the black holes' event horizons in dimensions greater than six. However, we do know of possible topologies: those realized by some explicitly known solutions. These include S^{D-2} (realized in the Myers-Perry solutions), $S^1 \times S^{D-3}$ (realized in the approximate solutions of thin black rings [11]), and finally, $T^p \times S^{D-2-p}$ with $p \geq 2$ (realized in the black p-tuboids). There is also evidence of existence of black holes with $\Pi_i S^{m_i} \times S^{D-2-m}$ horizon topologies where $2 \leq m_i \leq D-4$, where $m_i \in N_{odd}$ and $m = \Sigma_i m_i \leq D-4$. Collectively black holes with horizons that are products of spheres and tori (particularly dubbed black p-tuboids) are called blackfolds. Note that even-ball blackfolds are claimed to describe the ultra-spinning Myers-Perry black holes.

Much less is known about black hole solutions in curved backgrounds, with non-vanishing cosmological constant Λ. The reason is to be put down to the extra term that arises from the non-vanishing cosmological constant in the Einstein equation, which further complicates the problem. For instance in global Anti-de Sitter space, AdS, at spatial infinity, only AdS black holes with spherical horizons S^{D-2}, both static and stationary, *thin* AdS black ring and *thin* AdS black Saturn [12] in all dimensions greater than four have been found. Note that by topological censorship a four dimensional AdS/dS black ring can be ruled out.

New features of black objects

As the number of D space-time dimensions increases, so does the difficulty in solving Einstein's equations. Indeed, the larger number of degrees of freedom, $\frac{1}{2}(D-2)(D-1)-1$, carried by the unknown metric $g_{\mu\nu}$ to be solved for makes these equations an increasingly involved system of coupled, nonlinear, partial differential equations. In spite of this drawback, the D≥5-dimensional black hole solutions exhibit exciting new features which are worth studying.

But before plunging into the different new properties of the solutions let us go back to the field equations of the theory (in the vacuum) we will be interested in, namely GR in D-dimensions

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \frac{(D-2)}{2} \Lambda g_{\mu\nu} = 0 \quad \text{(a)}$$

($\mu,\nu = 1, 2, ..., D$ and Λ is the cosmological constant). Note despite their remarkable simplicity, they nevertheless hide extraordinary mathematical complexity. The geometry of space-time is encoded in the metric $g_{\mu\nu}$, which features explicitly and within the Ricci tensor $R_{\mu\nu}$ and scalar R that measures the curvature of space-time.

An investigation of black hole solutions of (a) in higher dimensions revealed several new intriguing aspects. Strikingly, certain black hole solutions of Einstein's equations (a) in higher dimensions, in contrast to their four dimensional cousins, have new features:

1) richer rotational dynamics: they can now rotate in up to N=[(D−1)/2] independent rotation planes, and the number of Casimir operators (independent angular momenta J_i^2) of the spacelike rotation group SO(D−1), with a fixed mass, have non constrained spins hence exhibit ultra-spinning regimes.

2) extended black hole solutions: these include black strings and black p-branes which can also carry charges and boosts and exhibit extended horizons with topologies $S^{D-2-p} \times R^p$.

3) multi horizon black holes: all the higher-dimensional black holes dealt with so far present a single event horizon and can, accordingly, be referred to as uni horizon black holes. However, unlike its four-dimensional counterpart, higher-dimensional GR also admits black-hole solutions with several, disconnected horizons: the so called multi horizon black holes. Examples of the simplest multi horizon back holes include a five-dimensional black Saturn [13], a combination of a black ring with a Myers-Perry black hole at its centre, the di-ring, consisting of two concentric co-planar black rings [14], and the bicycling black ring [15], consisting of two five

Figure 2. Black holes in higher dimensions

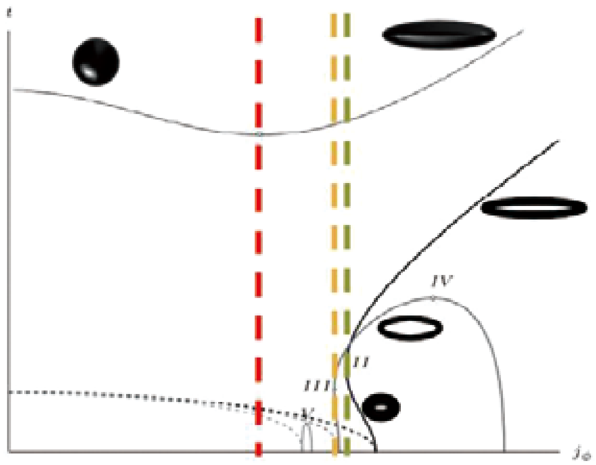

Figure 3. Plot of the temperature as a function of the angular momentum j_φ, for a fixed mass, for different black objects. These incude the singly spinning Myers-Perry black holes in five dimensions of space-time (*black dashed line*) and its seven dimensional cousin (*solid thin line*). The singly (*solid thick*) and double spinning black ring (*light gray*) for different values of angular momenta (*right towards left*) are also shown here. The red, yellow and green lines mark the critical/turning points where the black membrane behavior of black holes sets in.

dimensional black rings rotating in orthogonal planes (see Fig.2 for a representation of these possibilities).

A remarkable fact is, due to the richer rotational dynamics, there is a link between the properties of higher dimensional compact black holes that resemble those of the extended black objects in the ultraspinning regimes. Indeed it was observed that as one increases the angular momentum, the temperature of these ultra-spinning black holes reaches a minimum and then starts to grow as expected for the black membrane. Similarly, for the singly and doubly spinning black rings, as the angular momenta is increased the temperature has a turning point, rather than a minimum, that signals a change in the thermodynamics and the threshold of the ultra-spinning regime [16] where it can be approximated by a boosted black string (see Fig. 3). A qualitative understanding of this issue is related to the observation that when the mass is fixed, as the spin becomes large the event horizon spreads out in the plane of rotation: the spherical black holes become higher dimensional 'pancakes' approaching the geometry of a black branes and the black rings become thin and with a very large circle radius. As a consequence the ultraspinning black holes inherit the Gregory-Laflamme instability of the black strings and p-branes and are expected to be classically unstable.

To summarize, in spite of all this headway, the complete list of all possible topologies and new properties that the event horizon of a higher dimensional black hole can display is still unknown and yet a lot of work remains to be done. We expect that the new insights gained in this field help not only to deepen our understanding of the different species of regular black holes but also to make progress in current new lines of research including fluid dynamics, strongly coupled quantum field theory and condensed matter which rely on the fascinating subject of black holes in higher dimensions.

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