Mathematics in the Ancient and Middle Ages of Korea

The official history of Korean Mathematics starts from the Three Kingdoms period (1C B.C. - A.D. 10C). The three kingdoms consist of Goguryeo, Baekje and Silla. The Korean language has its own native numeric words such as Hana (one), Dul (two), On (hundred), Jeumeon (thousand), Dumeon (ten thousand) and so on and arithmetic was already being studied from the beginning of the Three Kingdoms period.

The Silla dynasty established a system of arithmetic education based on that of the Tang dynasty of China in A.D. 669, right after the unification of the three kingdoms and then absorbed the systems of the other two kingdoms, Goguryeo and Baekje.

The Silla system was based on learning centers, each equipped with an arithmetic scholar and an assistant. Students were aged between 15 and 30, irrespective of their status and they were educated for 9 years. They were appointed as Daenama (10th grade government position) after graduation. The qualifying examinations were based on knowledge of the books called Cheolsul, Samgae, Gujang and Yukjang. This system lasted until the Goryeo dynasty (A.D. 918- A.D. 1392).

Cheolsul was a book listed in the Tang dynasty's arithmetic education. It is called Zhui Shu in Chinese and translates to The Method of Interpolation. It is thought that the book dealt with the calculation of sums of infinite series, like calculating value of pi, by using inscribed and circumscribed polygons. But the book did not survive till the present. It has been recorded that the book was so difficult that few wanted to learn it. It is impressive that such a difficult subject that was forgotten in China was being studied in Korea during the Goryeo dynasty period.

Gujiang's Chinese title is Jiu Zhang which translates to The Nine Chapters on the Mathematical Art. It was a fundamental textbook of Eastern mathematics and its influence matches that of Euclid's Elements. While Western mathematics concentrated on logical deduction depending on the tradition of Euclid's Elements, Eastern mathematics developed focusing on algebra and so it solved even geometric problems in an algebraic way. Gujiang or Jiu Zhang contains practical problems about measuring rectangular and circular shaped land, proportional distributions, volume of hexahedrons and systems of quadratic equations.

Samgae and Yukjang were, however, not preserved and so we do not know their contents.

In the areas of geometry and number theory Greek mathematics takes precedence, notably the works of the Greek mathematician Diophantus (A.D. 3C), but Eastern mathematics was more advanced in the area of algebra.
Joseon and Japanese Mathematics

During the rule of King Sejong of the Joseon dynasty which came after the Goryeo dynasty, the arithmetic system was reorganized. This period is known as the golden age of Korean mathematics. According to Sejong Sillok (authentic record of King Sejong), Sanhak Gyojeonso (Office of Arithmetic Publishing) and Seupsanguk (Bureau of Learning Arithmetic) were established and many arithmetic textbooks were published. King Sejong himself learned arithmetic here. The curriculum was composed of five subjects: Sang Myeong San Beop, Yang Hwi San Beop, San Hak Gye Mong, O Jo San Gyeong and Ji San. The first, Sang Myeong San Beop, whose Chinese title is Xiang Ming Suan Fa, explains the elements of metrology, acoustics and computation of series. Yang Hwi San Beop, or Yang Hui Suan Fa in Chinese, contains problems about magic squares and geometric figures. San Hak Gye Mong, whose Chinese title is Suan Xue Qi Meng, deals with solutions of high degree equations. O Jo San Gyeong, with Chinese title Wu Cao Suan Jing, treats accounts, systems of equations and arithmetic series. Ji San, titled Di Suan in Chinese, is about measurements.

The study of arithmetic which blossomed during the Sejong era was damaged to a great extent by the Joseon-Japan war (1592-1598) and the Joseon-Qing war (1636-1637). In particular, a large number of important arithmetic books were lost. They were retrieved about a hundred years later during the rule of King Sukjong (1675-1720). While Joseon was striving to revive the system of arithmetic education, Japan tried to understand the arithmetic books that they took away from Joseon during the wars. Japanese arithmetic emerged in the name of Wasan about a hundred years after the wars.

Kowa Seki, the originator of Wasan, started studying arithmetic by transcribing Joseon’s books and he invented a symbolic algebra, which describes counting rods used in Tian Yuan Shu of San Hak Gye Mong (Suan Xue Qi Meng) as drawings. Tian Yuan Shu was the method of solving high degree polynomials and was the most outstanding achievement of Eastern mathematics. The same method was published as approximate solutions of high degree polynomials many centuries later in the 19th century by Homer, a British mathematician.

Joseon Mathematical Societies

There were three classes of Joseon’s arithmeticians or mathematicians: the gentry, the illuminists and the bureaucracy.

1) The gentry: their mathematics, based on the traditional view, was not different from metaphysical ideology. The scholar, Seok-Jeong Choi (1645-1715), wrote a mathematical book called Gu Su Ryak, which could be compared to the theological mathematics of the Roman philosopher Boethius (470-524). The Gu Su Ryak emphasizes that the origin of numbers is the first thing to learn and starts with the I Ching-like phrase “Numbers were born from Tao.” His system of numbers like Taiji, Yin-yang, Four Realms, etc. is related to I Ching notions.

2) The illuminists: the trend of thought pursued by the late Joseon scholars is called Silhak, the practical study. The scholars of Silhak evolved the thought of Sil-sa-gu-st (Seek truth from facts) and led an enlightenment movement. They tended to be encyclopedists and valued mathematics. For
example, Dae-Yong Hong (1731-1783) built a private astronomical observatory and asserted his own unconventional system of the universe. He discussed mathematics and its links to music, astronomy and calendrical calculation in his book *Ju Hak Su Yong*. Besides, he separated mathematics from philosophy and introduced the concept of infinity through the calculation of pi.

(3) The bureaucracy: this middle class was between the gentry and commoners. They were mathematicians in the bureaucratic system. The bureaucrat for arithmetic was called San-sa. He was appointed through a qualifying test and this system of appointment was unique to Joseon. The lists of successful candidates and assessments were recorded in *Ju Hak Ip Gyeok An*. There were 1627 San-sa between the late 15th century and the late 19th century who all belonged to families related to the arithmetic bureaucracy, with the exception of 205 of them.

The arithmetic bureaucrat Jeong-Ha Hong wrote *Gu Il Jip* (*Collection of Nine One*), which is composed of nine volumes in total and has three parts of three volumes each, entitled Heaven, Earth and Human being. According to his book, Hong competed with a Qing dynasty mathematician Guo-Zhu He. This is the only record of any international competition in Korean history. From this book we also know that *Suan Xue Qi Meng* was lost and *Tian Yuan Shu* was forgotten in China. (Even counting rods disappeared). Guo-Zhu He was extremely surprised to find that *Suan Xue Qi Meng* survived in Joseon. This find might have been a stimulus to the Qing dynasty. Shi-Lin Luo found three volumes of *Suan Xue Qi Meng*, printed by Joseon, in Liu Li Chang at Beijing and reprinted them. It was not known how the books came to be in Beijing, but he was very excited on finding them. He said that Eastern mathematics could not have been handed down without Joseon.

**Way to Modern Mathematics**

Although it was a somewhat later part of the last millennium, Joseon had a tendency to exclude *I Ching* philosophy from traditional mathematics and also ignore the mere practical aspects. In other words mathematics was thought of as an independent part of science, so it was not within the purview of ideological or practical mathematics.

Byeong-Gil Nam (1820-1869), a mathematician in the gentry class, wrote a book containing old problems selected from existing books. He researched their mathematical properties rather than practical problem solving methods. His purpose was to establish a systematic or formal framework of mathematics away from mere practical mathematics. He also studied with Sang-Hyeok Yi (1810-?), another middle class mathematician, the theory of equations and compared Western mathematics with Joseon mathematics. This shows that he was searching for a way towards modern mathematics. Above all Yi actively studied Western mathematics including infinite series through trigonometry.

Korean mathematics concentrated on the counting rod calculation and *Tian Yuan Shu* emphasizing the Eastern tradition. But the late Joseon was paving the way towards modern mathematics through independently developed calculations and trigonometry.